# II-2. EQUALIZATION OF WAVEGUIDE DELAY DISTORTION


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All-pass microwave networks can be realized by terminating the conjugate pair of arms of a wide-band 3-db coupler in identical reactive networks. For transmission in one direction, an ideal circulator can be used with one arm terminated in the reactive network (References 1, 2, and 3) (see Figure 1a and 1b). When the reactive network is a length of waveguide, tapered so that the cutoff wavelength decreases with distance from the input port, characteristics well suited for the equalization of the dispersive characteristics of waveguide sections result. A single linear tapered guide, in particular, is shown to be capable of reducing the time delay variation of a length of waveguide to less than 2 percent over the full operating frequency band of the waveguide. A family of design curves is presented giving the slope of the taper for a prescribed degree of equalization over a specified frequency range.


The phase properties of either circuit is given by the negative of the angle of the reflection coefficient of the taper. The reflection factor of a tapered waveguide is described by the nonlinear Riccatti differential equation. In general, this equation cannot be solved exactly. Existing approximate solutions to this equation are valid only when the reflection factor is small and slowly varying. In the present case, the structure is totally reflecting. Therefore, existing approximate solutions cannot be employed.

As a first approximation to the phase of the reflection factor of a tapered line, it was assumed that (1) the taper is sufficiently gradual so that there are no reflections along the propagating section of the taper, (2) total reflection occurs at the point along the taper at which the cross section is exactly the cutoff dimensions of the guide, and (3) the total reflection at the cutoff point of the tapered waveguide is that caused by an open circuit at that point along the taper.*

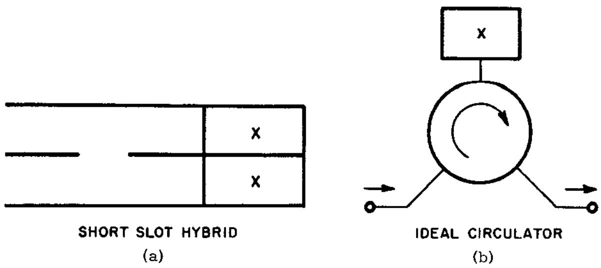


Figure 1. Microwave All-Pass Circuits

*A similar approach was taken independently by Tang in deriving the shape of a taper necessary to produce constant delay (Reference 2). The form of the solution reported here differs from that given by Tang. This new form leads to the wide-band equalization of waveguide runs by linear tapers.

The input reflection factor at the terminal plane T of Figure 2 is then equal to unity and has a phase angle $-\phi$, $\rho = e^{-j\phi}$. The phase shift introduced by such a section is $\phi = 2\theta$, where $\theta$ is the electrical length of the propagating section of the taper. On the basis of the foregoing assumptions, the electrical length of a differential segment of the taper is given by

$$d\theta = \frac{2\pi}{\lambda_g(x)}\, dx \tag{1}$$

where the guide wavelength $\lambda_g(x)$ is defined as the guide wavelength of a uniform waveguide whose cross section is identical to the cross section of the taper at x. Then the electrical length of a taper in the width of a rectangular guide is

$$\theta = 2\pi \int_0^{\ell} \frac{dx}{\lambda_g(x)} = 2\pi \int_0^{\ell} \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{a\lambda}\, dx \tag{2}$$

where $\lambda$ is the free space wavelength and $a = a(x)$ is the width of the waveguide at a distance x from the input terminal T. Assume a linear taper such that $a = a_o - kx$, where $a_o$ is the width of the input guide and k is constant. Then integrating from the taper input terminal to the cutoff point ($a = \lambda/2$) yields the phase shift

$$\phi = 2\theta = \frac{4\pi}{k\lambda}\left[\sqrt{a_o^2 - \left(\frac{\lambda}{2}\right)^2} - \frac{\lambda}{2}\cos^{-1}\frac{\lambda}{2a_o}\right]. \tag{3}$$

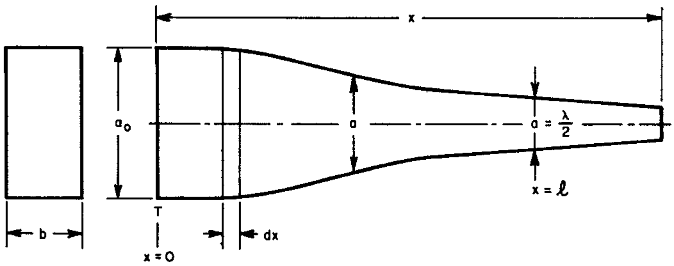


Figure 2. Tapered Rectangular Guide

Measured values of the reflection factor phase for a particular taper are compared to the phase predicted by Equation (3) in Figure 3. The almost constant phase difference between the two curves indicates that the termination of the taper at the point of cutoff is inductive, rather than the open circuit assumed in the derivation. This is in agreement with the characteristics of the $TE_{10}$ mode impedance below cutoff. The agreement in the slope of the phase characteristic demonstrates that Equation (3) can be used in the design of phase and time-delay equalizers with good accuracy.

The time delay, $d\phi/d\omega$, for this network was found to have the simple form

$$t_d = \frac{2a_o}{kc}\,\frac{\lambda}{\lambda_{go}} \tag{4}$$

where $\lambda_{go}$ is the guide wavelength in the input waveguide and c is the velocity of propagation in free space.

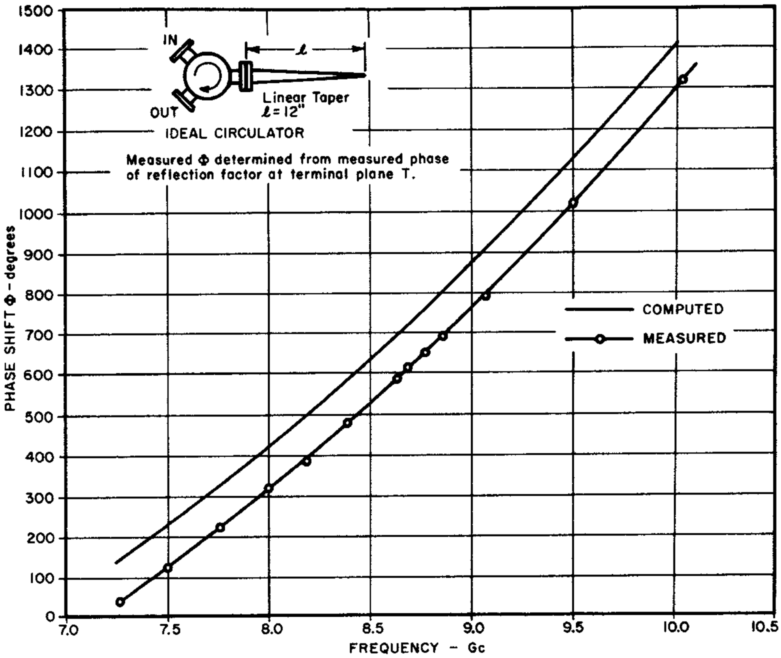


Figure 3. Phase Shift - Linear Tapered Equalizer

The total time delay of a length of waveguide L and an equalizer using a linearly tapered waveguide, with taper slope k, is given by

$$t_{d_t} = t_{d_w} + t_{d_E} = \frac{L}{c}\frac{\lambda_g}{\lambda} + \frac{2a_o}{kc}\frac{\lambda}{\lambda_g}, \tag{5}$$

where

$t_{d_t}$ = total time delay

$t_{d_w}$ = time delay of uniform waveguide of length L

$t_{d_E}$ = time delay of tapered equalizer,

The optimum correction will be obtained when $t_{d_t}$ exhibits a minimum in the operating band. The frequency at which minimum total time delay occurs is

$$f_m = f_c \sqrt{\frac{K}{K-1}}, \tag{6}$$

where

$K = \frac{2a_o}{kL}$; $K > 1$

$f_m$ = frequency of minimum total time delay

$f_c$ = cutoff frequency.

Conversely, if the frequency is specified at which the time delay is to be a minimum,

$$K = \frac{1}{1 - \left(\frac{f_c}{f_m}\right)^2} \quad \text{and} \quad k = \frac{2a_o}{L} \left[1 - \left(\frac{f_c}{f_m}\right)^2\right]. \tag{7}$$

The value of the time delay at its minimum is found from Equation (5) and Equation (6)

$$t_{d_{min}} = \frac{2L}{c} \sqrt{K} \tag{8}$$

and the relative change in time delay with respect to this minimum value is given by

$$\delta = \Delta t_d = \frac{t_d}{t_{d_{min}}} - 1 = \frac{1}{2\sqrt{K}} \left[ \frac{(+K) - K\left(\frac{f_c}{f}\right)^2}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \right] - 1 . \tag{9}$$

Equation (9) can be rewritten to express the two band edge frequencies at which the time delay variation $\delta$, is equal.

$$\left(\frac{f_{1,2}}{f_c}\right)^2 = \frac{K}{(K-1) - 2\delta(2+\delta) \pm 2(1+\delta)\sqrt{\delta(2+\delta)}} . \tag{10}$$

Equations (9) and (10) are plotted in Figure 4 for different values of the parameter K.

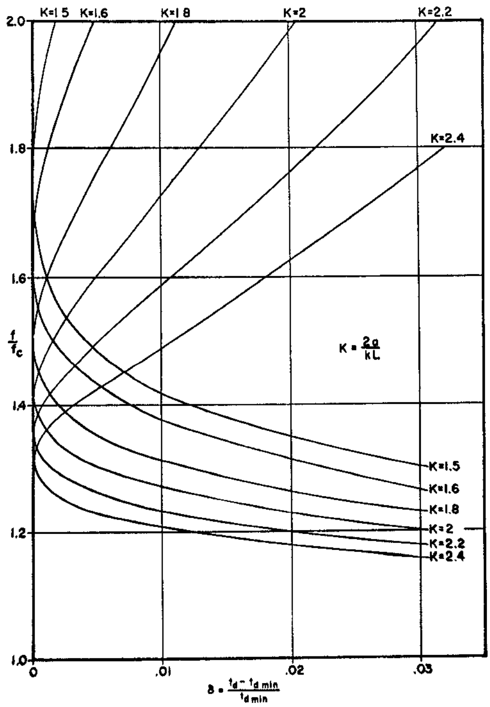


Figure 4. Design Data - Tapered Equalizer

A considerable degree of equalization can be achieved by the use of simple linearly tapered waveguide sections in conjunction with a circulator or broadband hybrid junction. The length of the taper required for the degree of equalization considered above is approximately half the length of the line to be equalized. Since the region of the taper close to cutoff contributes the greatest part of the delay correction, tapers having differently shaped transitions into the cutoff region will afford a greater economy of taper length without seriously affecting the degree of delay correction obtained. Shorter tapers will also result when equalization is required over narrower portions of the waveguide band. Greater equalization over the full band can be achieved by the cascade correction of a supplementary all-pass circuit of the type shown in Figure 1. The reactive network of the supplementary equalizer can be a simple resonant cavity.

## ACKNOWLEDGMENT

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## REFERENCES

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